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16. Abstract

The angular position and angular velocities at the end of the phase of active variation in angular velocity are defined with the aid of Euler angles (for the structural system of coordinates with respect to the immobile system), kinetic momentum, and differential equations.

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DETERMINATION OF THE ANGULAR POSITION AND ANGULAR VELOCITIES AT THE END OF THE PHASE OF ACTIVE VARIATION IN ANGULAR VELOCITY

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1. Statement of the Problem /177

Let us suppose that in an artificial satellite is placed a device which is capable at a given moment of turning it around some axis.

Let us call the phase of active variation in angular velocity that phase of motion during which this accelerating device operates.

Let us rigidly attach to the satellite a system of coordinates xyz, whose center coincides with the center of mass of the satellite. And the axes are directed along its main central axes of inertia. Since kinetic energy of rotation of the article rotated around the center of mass is greater than the work of external forces, the orbital motion of the satellite will not differ substantially from motion in a uniform field in which the vector $\bar{l}$ of kinetic momentum is constant. Let us introduce as an immobile system of coordinates the system $l_1 l_2 l_3$, associated with kinetic momentum. The origin of this system coincides with the center of mass of the object, c and the axis cl_3 is directed along vector $\bar{l}$. The position of the structural system of coordinates with respect to the immobile system will be defined with the aid of the Euler angles Ψ, Θ, Φ. For projections of the vector l on axis xyz we will have /178

$$\left.\begin{aligned} I_x\omega_x &= l \sin\Theta \sin\Phi, \\ I_y\omega_y &= l \sin\Theta \cos\Phi, \\ I_z\omega_z &= l \cos\Theta. \end{aligned}\right\} \quad (1)$$

Here $l = \sqrt{I_x^2\omega_x^2+I_y^2\omega_y^2+I_z^2\omega_z^2}$.

Projections onto the structural axes of angular velocity ω of rotation of the satellite around the center of mass have the form

$$\begin{aligned}\omega_x&=\dot{\Psi}\sin\Theta\sin\Phi+\dot{\Theta}\cos\Phi,\\ \omega_y&=\dot{\Psi}\sin\Theta\cos\Phi-\dot{\Theta}\sin\Phi,\\ \omega_z&=\dot{\Psi}\cos\Theta+\dot{\Phi}.\end{aligned} \qquad (2)$$

If by means of integrating the dynamic equations of Euler we will find the values of ω_x, ω_y, ω_z at the end of the phase of active variation of velocity, the values of the angles Ψ, Θ, Φ at that same moment will be derived from the relationships

$$\cos\Theta=\frac{I_z\omega_z}{l}, \qquad (3)$$

$$\sin\Phi=\frac{I_x\omega_x}{\sqrt{l^2-I_z^2\omega_z^2}}, \qquad \cos\Phi=\frac{I_y\omega_y}{\sqrt{l^2-I_z^2\omega_z^2}}, \qquad (4)$$

$$\Psi=\Psi_0+l\int_{t_0}^{t_k}\frac{2h-I_z\omega_z^2}{l^2-I_z^2\omega_z^2}\,dt, \qquad (5)$$

which are derived from (1) and (2). In formula (5) h denotes kinetic energy, i.e.,

$$2h=I_x\omega_x^2+I_y\omega_y^2+I_z\omega_z^2,$$

t_k denotes the turn-on time of the accelerating device.

2. Studying the Euler Equations

In the phase of active variation in angular velocity, due to the fact that the momentum of the accelerating device greatly exceeds the perturbing moments, the Euler equations of rotation of the article around the center of mass will be written as follows:

$$\left.\begin{aligned} \dot{\omega}_x+\omega_y\omega_z\frac{I_z-I_y}{I_x}&=0,\\ \dot{\omega}_y+\omega_x\omega_z\frac{I_x-I_z}{I_y}&=0,\\ \dot{\omega}_z+\omega_x\omega_y\frac{I_y-I_x}{I_z}&=M_p. \end{aligned}\right\} \tag{6}$$

Here M_p is the momentum of the accelerating device divided by I_z. /179

Given that $I_y = I_z = I$. Equations (6) acquire the form

$$\left.\begin{aligned} \omega_x&=\omega_x^0=\text{const},\\ \dot{\omega}_y+\omega_x^0\omega_z\frac{I_x-I}{I}&=0,\\ \dot{\omega}_z+\omega_x^0\omega_y\frac{I-I_x}{I}&=M_p. \end{aligned}\right\} \tag{7}$$

The solution of linear differential equations with constant coefficients (7) will be derived in the form

$$\left.\begin{aligned} \omega_y&=n^{-1}M_p+D_1\sin nt-D_2\cos nt-n^{-1}\int_0^t\frac{dM_p(\tau)}{d\tau}\cos n(t-\tau)\,d\tau,\\ \omega_z&=D_1\cos nt+D_2\sin nt+n^{-1}\int_0^t\frac{dM_p(\tau)}{d\tau}\sin n(t-\tau)\,d\tau. \end{aligned}\right\} \tag{8}$$

In formulas (8) the notation $n = \omega_x^0\frac{I - I_x}{I}$, is adopted; D_1 and D_2 are arbitrary constants of integration.

For simplicity, we can posit M_p = const. Formulas (8) are written thus:

$$\boxed{\begin{aligned} \omega_y&=n^{-1}M_p+D_1\sin nt-D_2\cos nt,\\ \omega_z&=D_1\cos nt+D_2\sin nt. \end{aligned}} \tag{9}$$

The constants of integration are easily calculated for given initial values ω_y^0 and ω_z^0. The solution of (8) has the form

$$\begin{aligned}\omega_y&=n^{-1}M_p(1-\cos nt)+\omega_z^0\sin nt+\omega_y^0\cos nt-n^{-1}\int_0^t\frac{dM_p(\tau)}{d\tau}\cos n(t-\tau)\,d\tau,\\ \omega_z&=\omega_z^0\cos nt+(n^{-1}M_p-\omega_y^0)\sin nt+n^{-1}\int_0^t\frac{dM_p(\tau)}{d\tau}\sin n(t-\tau)\,d\tau.\end{aligned} \tag{10}$$

For the case of solving (9), we will have expressions (10) in which the last terms on the right side are equal to zero.

Formulas (8)-(10) show that ω_z does not have a monotonous relationship as a function of time and that momentum M_p turns the article no only around axis cz, but also around axis cy. The mean velocity of rotating around axis cy is defined by the elementary formula of gyroscopic momentum:

$$M_{\text{гир}}=In\tilde{\omega}_y.$$

Since $M_{gyr}I^{-1} = M_p$, we have, as in formulas (8) and (9), $\omega_y = n^{-1}M_p$.

This examination shows that the equality of moments of inertia $I_y = I_z$ makes the construction of the article unsuited for operation. This same situation will occur when $I_y > I_z$.

The only working variant can be the case $I_y < I_z$. Study of this case should be done on a computer.

Equations (6) with constant M_p were integrated on an M-20 computer using the Runge-Kutta method, with automatic selection /180 of spacing under simulated initial conditions, presented in the following two versions:

$$
\begin{array}{lll}
1.\ \omega_x^0 = 1^0/\text{сек}, & \omega_y^0 = 3^0/\text{сек}, & \omega_z^0 = 6^0/\text{сек}, \\
2.\ \omega_x^0 = 0{,}2\ ^0/\text{сек}, & \omega_y^0 = 3^0/\text{сек}, & \omega_z^0 = 3^0/\text{сек}.
\end{array}
$$

Simulated values for M_p and moments of inertia of the satellite were used. The results of integration show that in each of the two versions ω_z increases virtually in proportion to time, attaining at the end of the phase of acceleration its highest value. The quantities $_x$ and $_y$ have fluctuations around the zero value. Where t_k ranges from 150 s to 250 s, integration requires about 3 minutes of computer time.

In predicting the rotary motion of an artificial satellite, after defining the mentioned angular velocities, the values of the Euler angles of the position of the structural system of the article with respect to the system associated with the vector of kinetic momentum can be calculated according to simple formulas (3), (4), (5).

Note: Calculation of a slight variation in moments of inertia of the satellite, which can be induced by escape of gases, can not essentially affect the conclusion formed above. To prove this assertion, let us examine the case $I_y = I_z = I$. Given that $n = \omega_x I^{-1}(I - I_x) = n_0 + \mu n'$, where $n_0 = \text{const}$, $n' = n'(t)$ and μ --a small parameter. Equations of orbital motion of a body of variable mass with reactive momentum having a projection along only axis cz in the form M_p will have [1] form (6); while for the case $I_y = I_z = I$--the form (7), respectively. On the basis of (7) we find that

$$
\begin{gathered}
\omega_y = n^{-1}(M_p - \dot{\omega}_z), \\
\ddot{\omega}_z + \dot{n}\omega_y + n^2\omega_z = \dot{M}_p.
\end{gathered}
\tag{11}
$$

The solution of equations (11) will be sought in the form of a series by steps μ and for the purpose of this investigation we will limit ourselves to calculation of terms on the order of μ. Let us denote by η and ξ the solution, respectively, for ω_y and ω_z where $\mu = 0$. The quantities η and ξ will coincide with ω_y and ω_z of formulas (10). Let us denote corrections of the first power by the prime.

Let us retain in equations (11) terms of the order μ. After transforming these equations with the aid of differential equations of zero approximation, we will find that

$$\begin{aligned} \omega'_y &= -n_0^2 n' M_0 - \dot{\omega}'_z, \\ \ddot{\omega}'_z + n_0^2 \omega'_z &= \dot{n}' \eta - 2 n_0 n' \xi. \end{aligned} \tag{12}$$

Equations (12) must be integrated with zero initial data. After integration and definition of the arbitrary constants, we will find that

$$\begin{aligned} \omega'_y &= n_0^{-2} \dot{n}' \eta (1 - \cos n_0 t) - n_0^{-1} \int_0^t f(\tau) \cos n_0 (t - \tau)\, d\tau, \\ \omega'_z &= n_0^{-1} \dot{n}' \eta \sin n_0 t + n_0^{-1} \int_0^t f(\tau) \sin n_0 (t - \tau)\, d\tau. \end{aligned} \tag{13}$$

In formula (13) the notation is adopted as follows:

$$f(\tau) = \dot{n}'(\tau)\, \eta(\tau) - 2 n_0 n'(\tau)\, \xi(\tau). \tag{14}$$

Let us multiply the solution of (13) by μ and sum it with the solution of zero approximation which appears as (10). After several transformations we will derive a representation for the /181 angular velocities accurate to within terms on the order μ in the following form:

$$\begin{aligned} \omega_y = n_0^{-1}(M_p + \mu n')(1 - \cos n_0 t) + \omega_y^0 \cos n_0 t - \\ - n_0^{-1} \int_0^t [\omega_z^0 - 2\mu n_0 n' \xi + \mu \dot{n}' \eta] \cos n_0 (t - \tau)\, d\tau, \end{aligned} \tag{15}$$

$$\begin{aligned} \omega_z = \omega_z^0 + (n_0^{-1} M_p - \omega_y^0 + \mu n_0^{-1} n' \eta) \sin n_0 t + \\ + n_0^{-1} \int_0^t \left[\frac{dM_p(\tau)}{d\tau} + \omega_z^0 + \mu \dot{n}' \eta - 2\mu n_0 n' \xi \right] \sin n (t - \tau)\, d\tau. \end{aligned} \tag{16}$$

According to the condition of the problem, the defining quantities of the right sides of formulas (15) and (16) are ω_z^0 and ξ, while in a comparatively small interval of time of acceleration, the quantity ξ can exceed ω_z^0 by several times (no more than 10). Formulas (15) and (16) show that under conditions of the examined problem, terms on the order of μ do not evoke great perturbations in the solution of zero approximation.

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